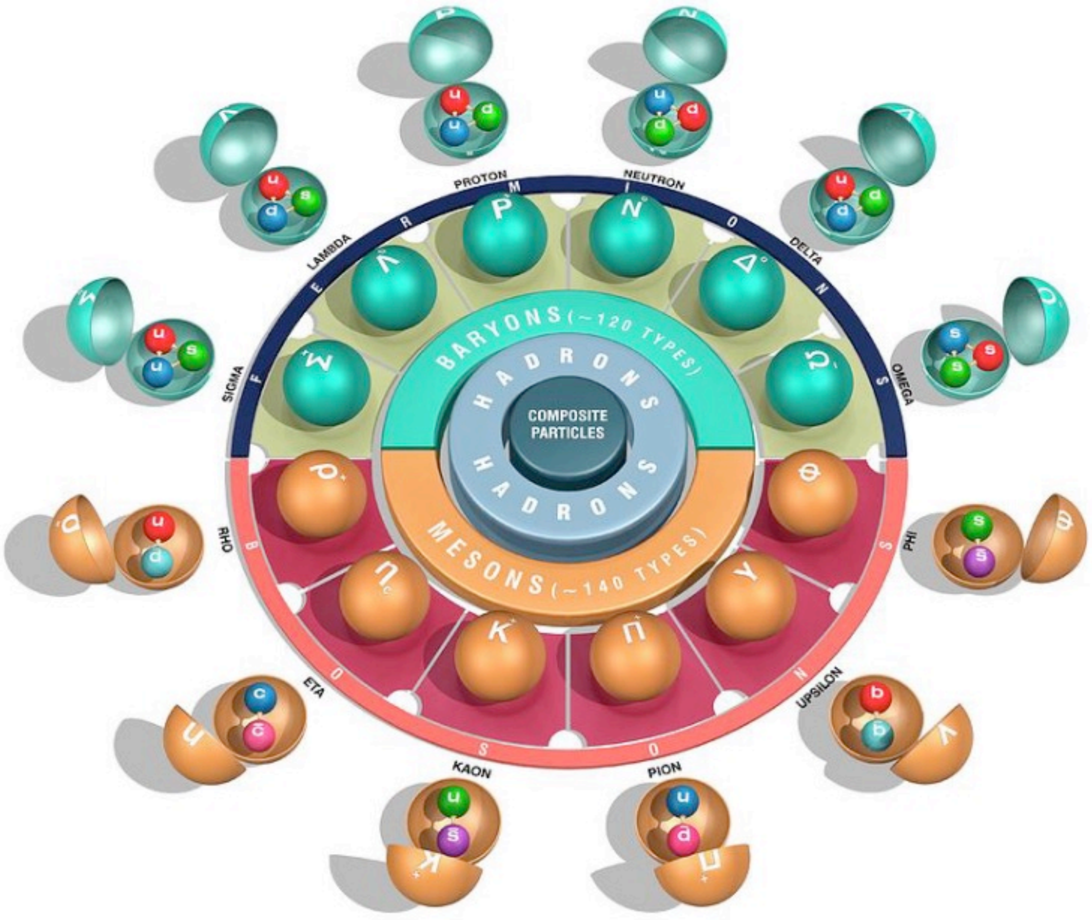

Particle Physics I

Lecture 11: Symmetries and the Quark Model

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Learning targets

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- Connection between symmetries and conservation laws
- Flavour symmetry of the strong interaction
- Introduction of the notion of isospin: definition, properties
- How to combine quarks into hadrons: baryons made of u and d quarks and baryon wavefunction

Introduction

- Symmetries play a fundamental role in particle physics – an aim of particle physics is to uncover the fundamental symmetries of the universe
- We will apply the idea of symmetry to the quark model with the aim to:
  - derive the hadron wave function
  - provide an introduction to the more abstract ideas of colour and QCD
  - ultimately explain why hadrons only exist as $q\bar{q}$ (mesons), qqq (baryons) or $\bar{q}\bar{q}\bar{q}$ (antibaryons)
- We will introduce ideas of the $SU(2)$ and $SU(3)$ symmetry groups which play a major role in particle physics

Symmetries and Conservation Laws

- Suppose physics is invariant under the transformation

$$\psi \rightarrow \psi' = \hat{U}\psi$$

e.g. rotation of the coordinate system

- It must conserve the probability normalisation condition

$$\langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \hat{U}\psi \middle| \hat{U}\psi \rangle = \langle \psi \middle| \hat{U}^{\dagger}\hat{U} \middle| \psi \rangle$$

$\Longrightarrow \hat{U}^{\dagger}\hat{U} = 1$ i.e. $\hat{U}$ must be a unitary operator

Symmetries and Conservation Laws

- For physical predictions to be changed by the symmetry transformation it is also required that all QM matrix elements remain unchanged

$$\langle\psi|\hat{H}|\psi\rangle = \langle\psi'|\hat{H}|\psi'\rangle = \langle\psi|\hat{U}^{\dagger}\hat{H}\hat{U}|\psi\rangle$$

which leads to the requirement

$$\hat{U}^{\dagger}\hat{H}\hat{U} = \hat{H}$$

$$\times\hat{U} \qquad \hat{U}\hat{U}^{\dagger}\hat{H}\hat{U} = \hat{U}\hat{H} \Longrightarrow \hat{H}\hat{U} = \hat{U}\hat{H}$$

$\Longrightarrow [\hat{H},\hat{U}] = 0$ i.e. $\hat{U}$ **must commute with the Hamiltonian**

Symmetries and Conservation Laws

- Now consider the infinitesimal transformation

$$\hat{U} = 1 + i\epsilon\hat{G}$$

$\hat{G}$ is called the generator of the transformation

- For $\hat{U}$ to be unitary:

$$\hat{U}^\dagger\hat{U} = \left(1 + i\epsilon\hat{G}\right)\cdot\left(1 - i\epsilon\hat{G}^\dagger\right) = 1 + i\epsilon\left(\hat{G} - \hat{G}^\dagger\right) + \mathcal{O}(\epsilon^2)$$

neglecting $\mathcal{O}(\epsilon^2)$: $\hat{U}^\dagger\hat{U} = 1 \Longrightarrow \hat{G} = \hat{G}^\dagger$ i.e $\hat{G}$ is a Hermitian operator and therefore corresponds to an observable quantity G!

- Furthermore: $\left[\hat{H}, \hat{U}\right] = 0 \Longrightarrow \left[\hat{H}, 1 + i\epsilon\hat{G}\right] = 0 \Longrightarrow \left[\hat{H}, \hat{G}\right] = 0$

- From QM: $\frac{d}{dt}\langle\hat{G}\rangle = i\langle\left[\hat{H}, \hat{G}\right]\rangle = 0 \Longrightarrow$ ***G* is a conserved quantity**

Symmetries $\Longleftrightarrow$ Conservation Laws

- Each symmetry in nature leads to an observable <u>conserved</u> quantity
- **Example:** infinitesimal spatial translation $x \rightarrow x + \epsilon$
  - we expect physics to be invariant (unchanged) under $\psi(x) \rightarrow \psi' = \psi(x+\epsilon)$
  - $\psi' = \psi(x+\epsilon) = \psi(x) + \frac{\partial \psi}{\partial x}\epsilon = \left(1 + \epsilon \frac{\partial}{\partial x}\right)\psi$
  - but $\hat{p}_x = -i\partial/\partial x \Longrightarrow \psi'(x) = (1 + i\epsilon\hat{p}_x)\psi(x)$
  - the generator of the symmetry transformation is $\hat{p}_x \Longrightarrow p_x$ is conserved
  - translation invariance of physics implies momentum conservation!
- In general, the symmetry operator may depend on more than one parameter

$$\hat{U} = 1 + i\vec{\epsilon} \cdot \vec{G}$$

- **Example:** for an infinitesimal 3D linear translation $\vec{r} \rightarrow \vec{r} + \vec{\epsilon} \Longrightarrow \hat{U} = 1 + i\vec{\epsilon} \cdot \vec{p}$, $\vec{p} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)$

Symmetries $\Leftrightarrow$ Conservation Laws

- So far, we only considered an infinitesimal transformation, however any finite transformation can be expressed as a series of infinitesimal transformations:

$$\hat{U}(\alpha) = \lim_{n\to\infty}\left(1 + i\frac{\vec{\alpha}}{n}\cdot\vec{G}\right)^n = e^{i\vec{\alpha}\cdot\vec{G}}$$

- **Example:** finite spatial translation in 1D $x \to x + x_0$ with $\hat{U}(x_0) = e^{ix_0\cdot\hat{p}_x}$:
  - $\psi'(x) = \psi(x + x_0) = \hat{U}\psi(x) = e^{(x_0 d/dx)}\psi,\quad$ with $\quad \hat{p}_x = -i\frac{\partial}{\partial x}$
  - $= \left(1 + x_0\frac{\partial}{\partial x} + \frac{x_0^2}{2}\frac{\partial^2}{\partial x^2} + \cdots\right)\psi(x)$
  - $= \psi(x) + x_0\frac{\partial\psi}{\partial x} + \frac{x_0^2}{2}\frac{\partial^2\psi}{\partial x^2} + \cdots$
  - we obtain the expected Taylor expansion

Symmetries in particle physics: isospin

- The proton and neutron have very similar mass, and the nuclear force is found to be (approximately) independent of charge

$$V_{pp} \approx V_{np} \approx V_{nn}$$

- To reflect this symmetry, Heisenberg proposed in 1932 that if one could "switch off" the electric charge of the proton, there would be no way to distinguish between a proton and a neutron
- He proposed that the proton and neutron should be considered as the two states of a single entity, **the nucleon**

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Analogous to the spin-up and spin-down states of spin-half particle, **called isospin**
- Physics is expected to be invariant under rotations in this space
- The neutron and proton form an isospin doublet with total isospin $I = 1/2$ and third component $I_3 = \pm 1/2$

Flavour symmetry of the strong interaction

- We can extend this idea to the quarks
- Assume that the strong interaction treats all quark flavours equally (it does)
  - because $m_u \approx m_d$ the strong interaction has an approximate flavour symmetry, from the point of view of the strong interaction nothing changes if all up quarks are replaced by down quarks and vice versa
- Choose the basis:

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Express the invariance of the strong interaction under $u \leftrightarrow d$ as invariance under "rotations" in the abstract isospin space

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \hat{U} \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

Flavour symmetry of the strong interaction

- The 2×2 unitary matrix depends on 4 complex numbers (8 real parameters) but we also get 4 constraints from $\widehat{U}^{\dagger}\widehat{U} = 1$

$$\Longrightarrow 8 - 4 = 4 \text{ independent matrices}$$

- In the language of group theory, these 4 matrices form the $\boldsymbol{U(2)}$ group

Flavour symmetry of the strong interaction

- One of the matrices corresponds to multiplying by a phase factor

$$\hat{U}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} e^{i\phi}$$

  not a flavour transformation and of no relevance here

- The remaining matrices form an $SU(2)$ subgroup (special unitary) with $\det U = 1$

- For an infinitesimal transformation, in terms of the Hermitian generators $\hat{G}$:

$$\hat{U} = 1 + i\epsilon\hat{G}, \qquad \det U = 1 \text{ and } \mathrm{Tr}\left(\hat{G}\right) = 0$$

- A linearly independent choice for $\hat{G}$ are the Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_1 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- The proposed flavour symmetry of the strong interaction has the same transformation properties as spin!

Flavour symmetry of the strong interaction

- Define isospin:

$$\vec{T} = 1/2\vec{\sigma}\,, \hat{U} = e^{i\vec{\alpha}\cdot\vec{T}}$$

- For an infinitesimal transformation

$$\hat{U} = 1 + \frac{i}{2}\vec{\epsilon}\cdot\vec{\sigma} = 1 + \frac{i}{2}(\epsilon_1\sigma_1 + \epsilon_2\sigma_2 + \epsilon_3\sigma_3) = \begin{pmatrix} 1 + i/2\cdot\epsilon_3 & i/2\cdot(\epsilon_1 - i\epsilon_2) \\ i/2\cdot(\epsilon_1 + i\epsilon_2) & 1 - i/2\cdot\epsilon_3 \end{pmatrix}$$

which is required by unitarity and has a unit determinant

$$\hat{U}^\dagger\hat{U} = I + \mathcal{O}(\epsilon^2), \qquad \det U = 1 + \mathcal{O}(\epsilon^2)$$

Properties of isospin

- Isospin has the same properties as spin

$$[T_1, T_2] = iT_3, \qquad [T_2, T_3] = iT_1, \qquad [T_3, T_1] = iT_2$$

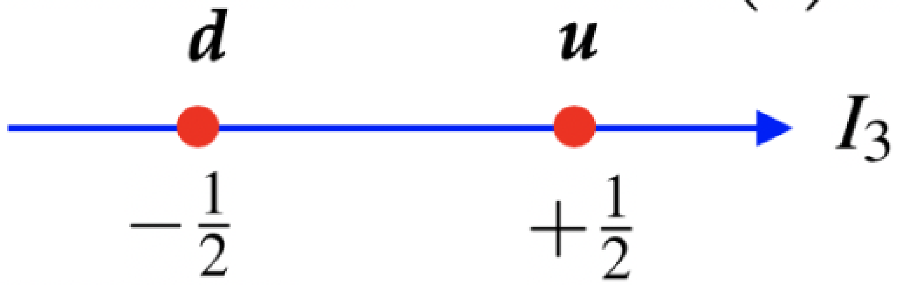
$$[T^2, T_3] = 0, \qquad T^2 = T_1^2 + T_2^2 + T_3^2$$

- Like in the case of spin, we have three non-commuting operators T_1, T_2, T_3 and even though all three correspond to observables we can't measure them simultaneously.
- We label states in terms of the total isospin I and the third component of the isospin I_3
- *Note:* physically isospin has nothing to do with spin – just the same algebra

Properties of isospin

- The eigenstates are exact analogues to the eigenstates of ordinary angular momentum $|l, m\rangle \rightarrow |I, I_3\rangle$ with $T^2|I, I_3\rangle = I(I+1)|I, I_3\rangle$ and $T_3|I, I_3\rangle = I_3|I, I_3\rangle$

$$u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\tfrac{1}{2}, +\tfrac{1}{2}\rangle \qquad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\tfrac{1}{2}, -\tfrac{1}{2}\rangle$$



$$\boxed{I = \frac{1}{2}, \quad I_3 = \pm\frac{1}{2}}$$

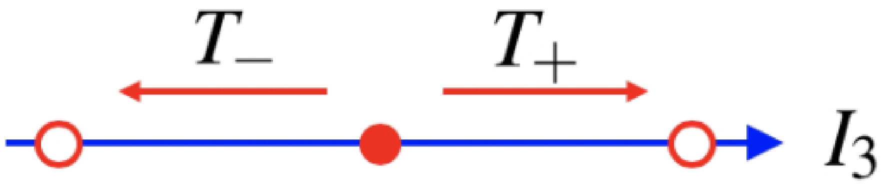
- In general: $I_3 = \frac{1}{2}(N_u - N_d)$

Properties of isospin

- We can define the isospin ladder operators – analogous to spin ladder oerators

$$\boxed{T_- \equiv T_1 - iT_2}$$

u → d



$$\boxed{T_+ \equiv T_1 + iT_2}$$

d → u

$$T_+|I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3+1)}\,|I, I_3+1\rangle$$
$$T_-|I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3-1)}\,|I, I_3-1\rangle$$

- Go up and down in I_3 until you reach the end of the **multiplet**: $T_+|I, +I\rangle = 0$ and $T_-|I, -I\rangle = 0$

$$T_+u = 0, \qquad T_+d = u, \qquad T_-d = 0, \qquad T_-u = d$$

- Ladder operators turn u into d and d into u

Properties of isospin

- **Combination of isospin:** the isospin of a system of two d quarks is exactly analogous to combination of spin (or angular momentum)

$$\left|I^{(1)}, I_3^{(1)}\right\rangle \left| I^{(2)}, I_3^{(2)}\right\rangle \rightarrow |I, I_3\rangle$$

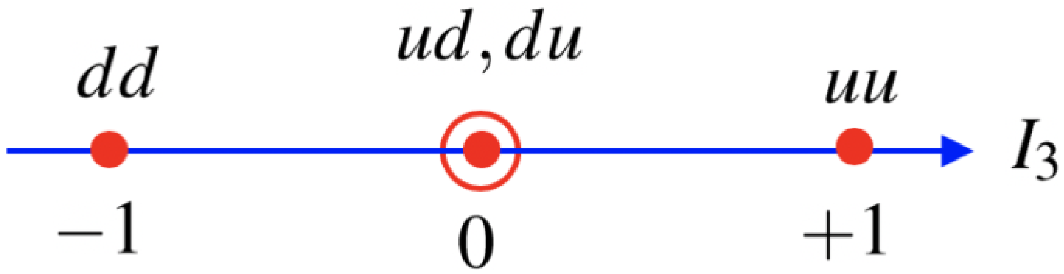
- I_3 is additive: $I_3 = I_3^{(1)} + I_3^{(2)}$
- I is in integer steps from $\left|I_3^{(1)} - I_3^{(2)}\right|$ to $\left|I_3^{(1)} + I_3^{(2)}\right|$

Properties of isospin

- The assumed symmetry of the strong interaction under isospin transformations implies the existence of conserved quantities
- In strong interactions I_3 and I are conserved, analogous to conservation of J_z and J for angular momentum
- It's a **natural conclusion** because the strong interaction does not change the flavour and charge of quarks!

Combining quarks to form hadrons

- **Goal:** derive the proton wavefunction
  - first combine two quarks, then add a third
  - use the requirements that **fermion wavefunctions are antisymmetric**
- Isospin starts to become useful in defining states with more than one quark
- For two quarks there are four possible combinations:



Note: represents two states with the same value of I_3

Combining quarks

- We can immediately identify the extremes (I_3 additive):

$$uu \equiv \left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle = |1,+1\rangle, \qquad dd \equiv \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle = |1,-1\rangle$$

- To obtain the $|1,0\rangle$ state we use the ladder operators:

$$\mathrm{T}_-\left|1,+1\right\rangle = \sqrt{2}\left|1,0\right\rangle = T_-(uu) = ud + du$$

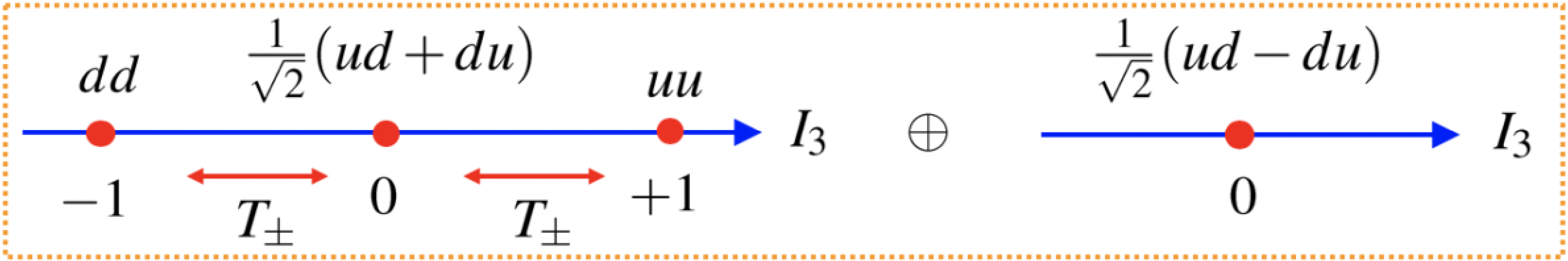
$$\Rightarrow |1,0\rangle = \frac{1}{\sqrt{2}}(ud + du)$$

- The final state $|0,0\rangle$ can be found from orthogonality with $|1,0\rangle$

$$|0,0\rangle = \frac{1}{\sqrt{2}}(ud - du)$$

Combining quarks

- From the four possible combinations of isospin doubles obtain a triplet of isospin-1 states and a singlet isospin 0 state: $2 \otimes 2 = 3 \otimes 1$



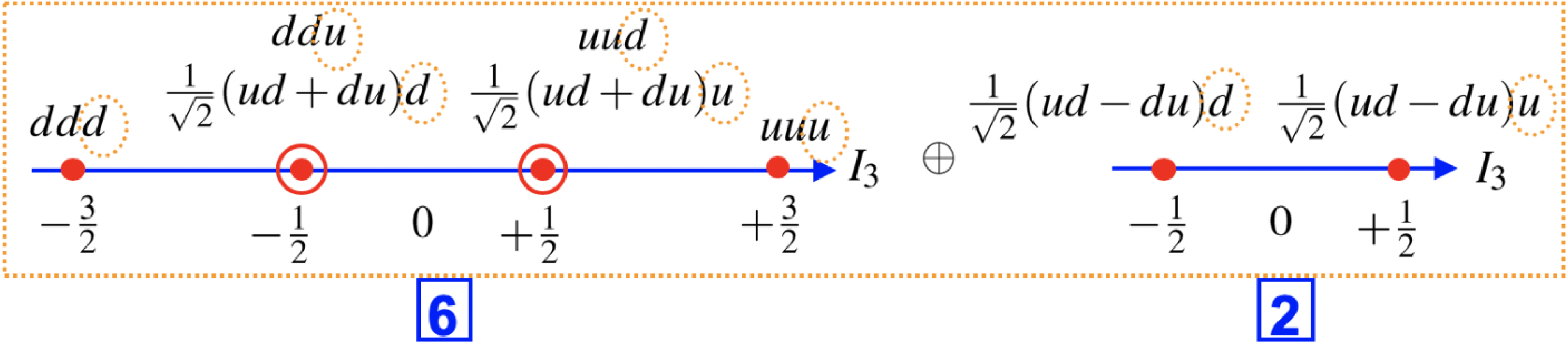
- We can move within multiplets using ladder operators

- *Note:* as anticipated $I_3 = \frac{1}{2}(N_u - N_d)$

- States with different isospin are physically different – the **isospin-1 triplet** is **symmetric** under interchange of quarks 1 and 2 whereas the **singlet** is **antisymmetric**

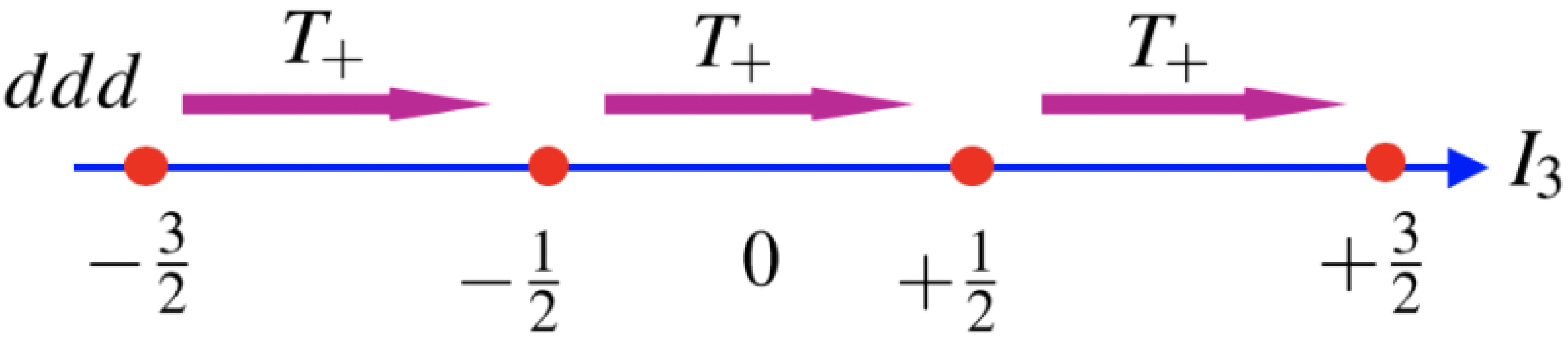
Combining quarks

- Now add an additional up or down quark
- From each of the above four states we get two new isospin states with $I_3' = I_3 \pm \frac{1}{2}$



Combining quarks

- Use ladder operators and orthogonality to group the 6 states into isospin multiplets
- Obtain the $I = 3/2$ states, step up from ddd



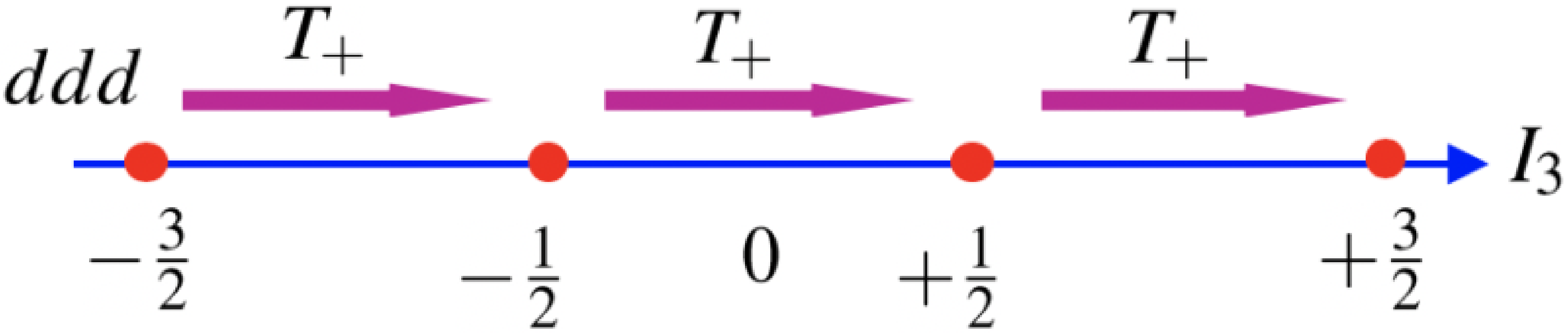
$$T_+|3/2, -3/2\rangle = T_+(ddd) = (T_+d)dd + d(T_+d)d + dd(T_+d)$$

$$\sqrt{3}\,|3/2, -1/2\rangle = udd + dud + ddu$$

$$\Longrightarrow |3/2, -1/2\rangle = 1/\sqrt{3}(udd + dud + ddu)$$

Combining quarks

- Use ladder operators and orthogonality to group the 6 states into isospin multiplets
- Obtain the $I = 3/2$ states, step up from ddd



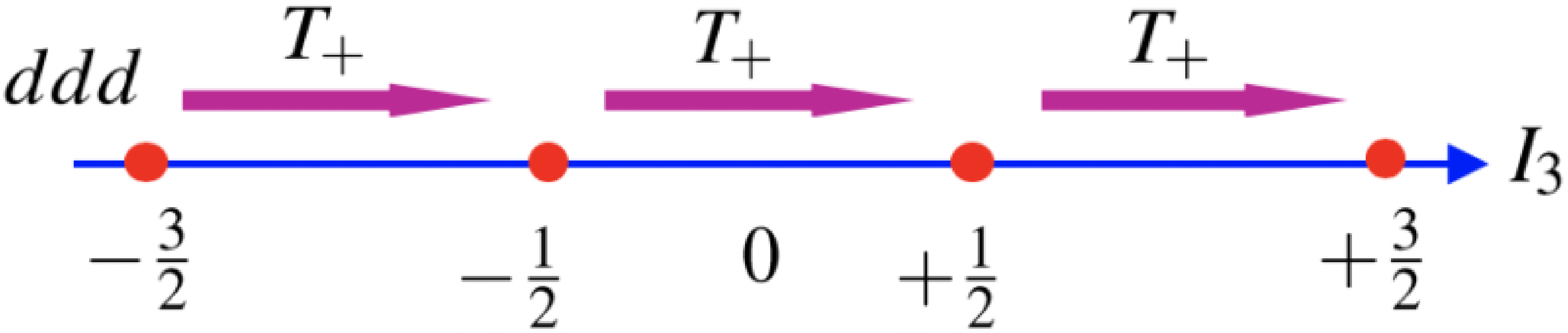
$$T_+|3/2,-1/2\rangle = 1/\sqrt{3}T_+(udd + dud + ddu)$$

$$2\,|3/2,+1/2\rangle = (uud + udu + uud + duu + udu + duu)$$

$$\Longrightarrow |3/2,+1/2\rangle = 1/\sqrt{3}(uud + udu + duu)$$

Combining quarks

- Use ladder operators and orthogonality to group the 6 states into isospin multiplets
- Obtain the $I = 3/2$ states, step up from ddd



$$T_+|3/2,+1/2\rangle = 1/\sqrt{3}T_+(uud + udu + duu)$$

$$2\,|3/2,+3/2\rangle = 1/\sqrt{3}(uuu + uuu + uuu)$$

$$\Longrightarrow |3/2,+3/2\rangle = uuu$$

Combining quarks

- From the six states we used orthogonality to find the $|1/2, \pm 1/2\rangle$
- The eight states $uuu, uud, udu, udd, duu, dud, ddu, ddd$ are grouped into an isospin quadruplet and two isospin doublets

$$2 \otimes 2 \otimes 2 = 2 \otimes (3 \otimes 1) = (2 \otimes 3) \otimes (2 \otimes 1) = 4 \otimes 2 \otimes 2$$

- Different multiplets have different symmetry properties:
    - S: a quadruplet of states which are symmetric under the interchange of any two quarks
    - M_S: a doublet of mixed symmetry, symmetric for $1 \leftrightarrow 2$
    - M_A: a doublet of mixed symmetry, antisymmetric for $1 \leftrightarrow 2$
- Mixed-symmetry states have no definite symmetry under interchange of quarks $1 \leftrightarrow 3$ etc.

Combining quarks: summary

$$\left.\begin{aligned}|\tfrac{3}{2},+\tfrac{3}{2}\rangle &= uuu\\ |\tfrac{3}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(uud+udu+duu)\\ |\tfrac{3}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(ddu+dud+udd)\\ |\tfrac{3}{2},-\tfrac{3}{2}\rangle &= ddd\end{aligned}\right\}\ \boxed{\mathbf{S}}$$

A quadruplet of states which are symmetric under the interchange of <u>any</u> two quarks

$$\left.\begin{aligned}|\tfrac{1}{2},-\tfrac{1}{2}\rangle &= -\tfrac{1}{\sqrt{6}}(2ddu-udd-dud)\\ |\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{6}}(2uud-udu-duu)\end{aligned}\right\}\ \boxed{\mathbf{M_S}}$$

Mixed symmetry.
Symmetric for 1 $\longleftrightarrow$ 2

$$\left.\begin{aligned}|\tfrac{1}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{2}}(udd-dud)\\ |\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{2}}(udu-duu)\end{aligned}\right\}\ \boxed{\mathbf{M_A}}$$

Mixed symmetry.
Anti-symmetric for 1 $\longleftrightarrow$ 2

Combining spin

- We can use exactly the same mathematics to determine the possible spin wavefunctions for a combination of 3 spin-half particles

$$\left.\begin{aligned}
|\tfrac{3}{2},+\tfrac{3}{2}\rangle &= \uparrow\uparrow\uparrow \\
|\tfrac{3}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \\
|\tfrac{3}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow) \\
|\tfrac{3}{2},-\tfrac{3}{2}\rangle &= \downarrow\downarrow\downarrow
\end{aligned}\right\} \boxed{\mathbf{S}}$$

A quadruplet of states which are symmetric under the interchange of any two quarks

$$\left.\begin{aligned}
|\tfrac{1}{2},-\tfrac{1}{2}\rangle &= -\tfrac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \\
|\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)
\end{aligned}\right\} \boxed{\mathbf{M_S}}$$

Mixed symmetry. Symmetric for 1 $\longleftrightarrow$ 2

$$\left.\begin{aligned}
|\tfrac{1}{2},-\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow - \downarrow\uparrow\downarrow) \\
|\tfrac{1}{2},+\tfrac{1}{2}\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)
\end{aligned}\right\} \boxed{\mathbf{M_A}}$$

Mixed symmetry. Anti-symmetric for 1 $\longleftrightarrow$ 2

Baryon wavefunctions $(\boldsymbol{ud})$

- Quarks are fermions $\Longrightarrow$ require that the total wavefunction is antisymmetric under the exchange of any two quarks
- The total wavefunction can be expressed in terms of

$$\psi = \phi_{\text{flavour}} \chi_{\text{spin}} \xi_{\text{color}} \eta_{\text{space}}$$

- The color wavefunction for all bound qqq states is antisymmetric (not a subject of this lecture)
- Here we will only consider the lowest mass, ground-state baryons with no internal orbital angular momentum
- For $L = 0$ the spatial wavefunction is symmetric:$(-1)^L$

$\Rightarrow$ $\xi_{\text{colour}} \eta_{\text{space}}$ **anti-symmetric**

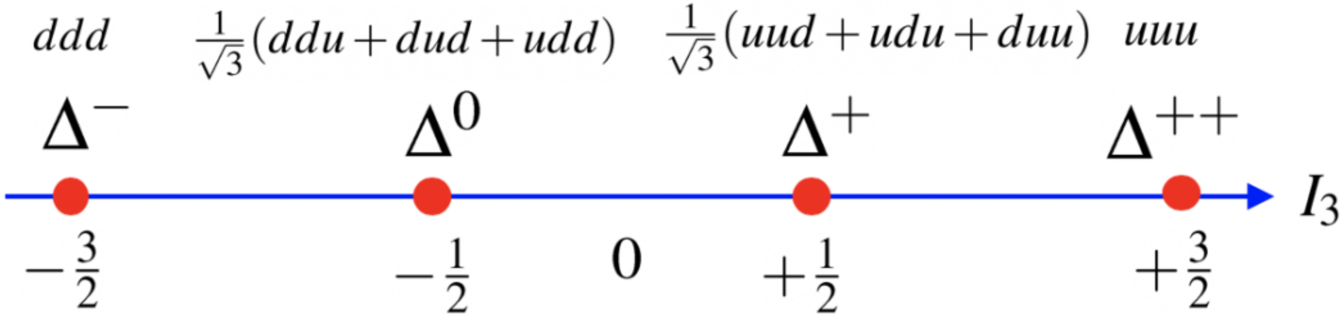
$\Rightarrow$ $\phi_{\text{flavour}} \chi_{\text{spin}}$ **symmetric**

Overall anti-symmetric

Baryon wavefunctions ($\boldsymbol{ud}$)

- Two ways to form a totally symmetric wavefunction from spin and isospin states

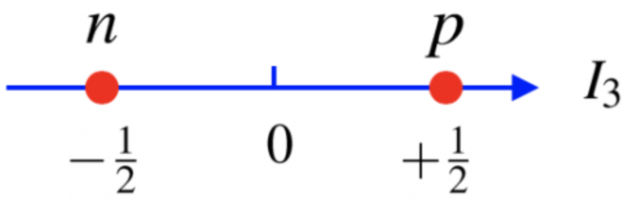
1. Combine totally symmetric spin and isospin wavefunctions $\phi(S)\chi(S)$



Spin 3/2
Isospin 3/2

2. Combine mixed–symmetry spin and mixed–symmetry isospin states

   - both $\phi(M_S)\chi(M_S)$ and $\phi(M_A)\chi(M_A)$ are symmetric under interchange of quarks $1 \leftrightarrow 2$
   - not sufficient, these combinations have no definite symmetry under interchange of quarks $1 \leftrightarrow 3$
   - it can be shown that the normalised linear combination is totally symmetric (under $1 \leftrightarrow 2$, $1 \leftrightarrow 3$, and $2 \leftrightarrow 3$)



Spin 1/2
Isospin 1/2

$$\frac{1}{\sqrt{2}}\big(\phi(M_S)\chi(M_S)+\phi(M_A)\chi(M_A)\big)$$

Baryon wavefunctions ($\boldsymbol{ud}$)

- The spin-up proton wavefunction is therefore

$$|p\uparrow\rangle = \frac{1}{6\sqrt{2}}(2uud - udu - duu)(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) + \frac{1}{2\sqrt{2}}(udu - duu)(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow)$$

$$\Rightarrow \boxed{\begin{aligned}|p\uparrow\rangle = \frac{1}{\sqrt{18}}(\ & 2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow + \\ & 2u\uparrow d\downarrow u\uparrow - u\uparrow d\uparrow u\downarrow - u\downarrow d\uparrow u\uparrow + \\ & 2d\downarrow u\uparrow u\uparrow - d\uparrow u\downarrow u\uparrow - d\uparrow u\uparrow u\uparrow)\end{aligned}}$$

- *Note:* the fully symmetric proton wavefunction would include the antisymmetric color wavefunction , which itself has six terms, giving a total of 54 terms with different combinations of flavour, spin and color. In practice, the above proton wavefunction is sufficient to calculate the physical properties of the proton (e.g. magnetic moment)

Summary of Lecture 11

<u>Main learning outcomes</u>

- What is the connection between symmetries and conservation laws in particle physics
- Isospin properties and algebra
- How to combine u and d quarks into hadrons
- Baryon wavefunction